A simple tool for calculating egg shape, volume and surface area from digital images

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The geometrical properties of eggs – such as volume and surface area – have uses ranging from ecological, physiological and morphological studies in birds, to predictions of chick condition in the poultry industry. Although measurements of an egg's length and breadth can be used to approximate an egg's geometry, the coefficients used in these models are specific to the original test population, and intraspecific variation in egg shape means these methods cannot be used reliably outside of that original test population. Here I present a novel mathematical formula to describe the curvature of a bird's egg that can be used to calculate the shape, volume and surface area of an egg precisely from digital images. Using data from a number of species I demonstrate that the model has a greater level of accuracy than length and breadth-based methods, and release the user-friendly tool for others to use for measuring eggs from digital images.

Keywords: egg metrics, field techniques, museum techniques, digital photography, egg measurement, Great tit, *Parus major* , Blue tit, *Cyanistes caeruleus,*

Egg size can be used to predict a number of important variables in avian ecology. Often used as a proxy for female investment in reproduction, larger eggs hatch into heavier chicks that are more likely to survive (Boersma, 1982, Narushin *et al.*, 2002, Reid & Boersma, 1990, Williams, 1994). Egg mass can be used as a reliable predictor of size, however, water loss gradually changes the eggs' mass over time, limiting the method to fresh eggs. Volume is a more reliable predictor of an egg's size, but measuring this precisely can be problematic. Bird eggs vary in their shape substantially within and between species, making simple geometrical rules for estimating egg volume or surface area difficult to generalise (Boersma & Rebstock, 2010, Bridge *et al.*, 2007). Measurements of egg length and breadth have commonly been used to estimate volume, particularly Hoyt's (1979) equation that incorporates length, breadth, and a species-specific shape variable (Volume $\approx$ 0.51 x Length x Breadth^2). However, the use of this model fails to account for intraspecific egg shape variation, and makes the assumption that the eggs being measured match the shape of the original test population.

Photography presents a number of advantages when calculating egg metrics. Digital cameras are cheap and ubiquitous, eggs can be photographed rapidly and easily *in situ* with low risk of breakage, and in conditions where volume cannot be measured based on air/water weight differentials (e.g. Boersma & Rebstock, 2010), eggs can be measured irrespective of age and water loss (including the use of museum collections). Furthermore a photographic archive of the eggs is recorded that can be used for additional purposes, such as visual modelling. A number of photographic techniques have been presented previously (Bridge *et al.*, 2007, Mänd *et al.*, 1986, Mónus & Barta, 2005, Paganelli *et al.*, 1974, Redondo & Arias-de-Reyna, 2002), nevertheless these techniques are either difficult to implement without further user programming/model fitting, or the studies failed to test their egg volume estimates against real egg volumes. Bridge *et al.* (2007) present an automated method for calculating egg metrics, however, the authors failed to test the model's accuracy against real egg volumes, instead comparing their values to Hoyt-based estimates. Moreover, the method presented by Bridge et al. requires egg photographs taken against high contrast backgrounds so that the automated software can split the egg into slices. Such automated processes that use thresholding present difficulties when attempting to photograph eggs that contain high contrast patterning that could interfere with the egg's perceived outline, or under non-diffuse lighting situations.

Egg shape fitting methods to calculate egg size are advantageous because they do not rely on image thresholding, however, these methods often require a large number of measurements of an egg's width at specific distances (e.g. Mónus & Barta, 2005, Smart, 1991), making them time consuming, or require specialist equipment, and to date there is no simple tool for fitting egg-shaped curves from digital images. Current egg shape models are able to match the egg shapes commonly found in nature (reviewed by Smart, 1991), however I set out to create a biologically inspired mathematical model of an egg shape that can: i) be solved easily by a least-squares method and ii) contain as few variables as possible to minimise the number of anchor points required for fitting.

METHODS

Model Development

Egg shape is thought to derive from pressures exerted in the oviduct, without which an elliptical egg would be formed. I therefore assumed that this pressure was exerted in a normal distribution along the length of the egg, generating an equation for an ellipse that incorporates a normal distribution, the mean and variance of which can be adjusted to fit the pointedness of an egg. For a given point along an egg's length *l*, it's radius *r* can be described thus:

$$r=\frac{a e^{\frac{-l^2}{2b^2}+\frac{cl}{b^2}-\frac{c^2}{2b^2}}\sqrt{1-l}\sqrt{l}}{\pi b}$$

Equation 1.

where *a* specifies the overall width of the egg, *b* represents the spread of the distribution (equivalent to σ in a normal distribution), and *c* specifies the location of the distribution's peak along the egg's length (equivalent to μ in a normal distribution); see figure 1 for egg shape examples. A least-squares function is then used to fit the model to the egg edges, requiring anchor points in the image selected at the base and tip of the egg and a minimum of three additional anchor points on each side of the egg, although any number of points can be used. Once the curvature of an egg is modelled, volume and surface area can be calculated by assuming a circular cross-section and splitting the egg into a large number of long-axis slices (10,000). Surface area is calculated by splitting each slice into 1,000 radial sections and summing the surface area of the trapezoids created by these projections, meaning surface area is calculated from 10^7 flat faces. Here I use a range of different species' eggs to test how accurately and precisely this method can calculate egg volume.

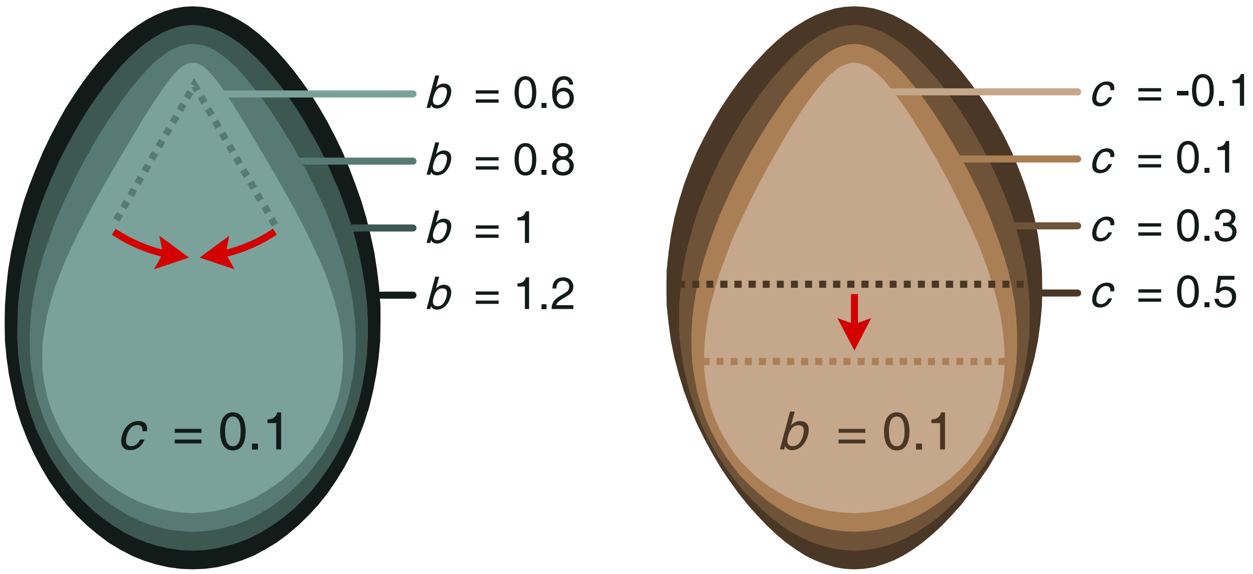

Figure 1. Model egg shapes: Varying coefficients *b* and *c* in the egg model (equation 1) can produce a wide range of egg shapes. Changing *b* alters the spread of the egg shape, with smaller numbers creating a greater 'pinching' effect (e.g. left). Changing *c* shifts the location of the egg bulge, from one end of the egg (e.g. right) to the centre, creating a symmetrical egg with values equal to 0.5. Changing *a* uniformly alters the overall width of the egg (not shown).

Model Testing

The quality of the model's fit to a wide range of egg shapes was assessed by testing the model on digital photographs of Great tit *Parus major* (n=10), Blue tit *Cyanistes caeruleus* (n=10), Crowned plover *Vanellus coronatus* (n=6), African wattled lapwing *Vanellus senegallus* (n=4), Japanese quail *Coturnix japonica* (n=30), Domesticated duck *Anas platyrhynchos domesticus* (n=12), Domesticated goose *Anser anser domesticus* (n=1), and Chicken *Gallus gallus domesticus* (n=30) eggs, with a large number of anchor points positioned precisely around the edge of the eggs (48 points per egg). The taxa chosen represent differing shapes, such as the “pointed” eggs of the ground-nesting plovers that are notoriously difficult to model together with easily modelled elliptical eggs of the cavity-nesting tits, and highly variable shapes of quail eggs. The coefficient of determination (R^2 value) of the least-squares fit was recorded to gauge quality of fit. These models were then compared to a model that used just eight anchor points to verify the use of a more user-friendly number of anchor points, one anchor was positioned at the tip, one at the base, and the remainder evenly down each side. (as shown in figure 2a). The accuracy of the model in calculating egg volume from digital photographs was assessed by comparing estimates from the digital image model with measured egg volumes. The volumes of quail (n=30), duck (n=12) and chicken (n=30) eggs were determined by comparing their weight in air (at *c.* 15m above sea level) with their weight submerged in water. All eggs were fresh (having a negative buoyancy), and the date stamping on duck and chicken eggs ensure none could have been laid by the same female, although quail egg lay dates could implicate a minimum of 15 females amongst the 30 eggs.. Photographs were taken at an angle of 90 degrees to the eggs' long axis with a Canon 5D Mark 2 camera and Canon 24-105 L lens, a ruler was used in the images as a scale bar, placed level with the centre of the eggs to ensure accuracy. No adjustments for lens distortion were deemed necessary in these photographs, though doing so could increase the accuracy of the results. Length and breadth measurements of the eggs were made with Mitutoyo digital callipers to a precision of 0.01mm in order to compare volume estimates with Hoyt-based estimates. Hoyt equation egg shape coefficients for each species were generated from the egg volumes measured for this study using a custom-written least squares approach in ImageJ. This ensures the Hoyt-based estimates are as close as they can possibly be to the real volume estimates and any error cannot result from differences between the egg shapes used in this study and those originally published by Hoyt. Statistics were performed in R (version 2.15.2 (R Core Team, 2012)). Photograph-based volume estimates and Hoyt-based volume estimates were compared to measured volumes using two-tailed paired t-tests with 95% confidence intervals. Egg pointedness was calculated as the average deviation in cross-sectional shape from an ellipse (sum of squares), where l is a point along the egg's long axis and r is the radius of the egg at length l ; r_{max} is the maximum radius of the egg, and n is the number of points measured along the egg's length (see figure 2c):

$$Pointedness = \sum_{l=1}^{l=0} \frac{\left(r_{max}\left(2\sqrt{(1-l)}\sqrt{l}\right)-r_l\right)^2}{n}$$

Equation 2.

The effect of pointedness on error in volume estimates was assessed using linear models that included species as a fixed effect and pointedness as a covariate. Data were checked for normality of error structure and homogeneity of variance.

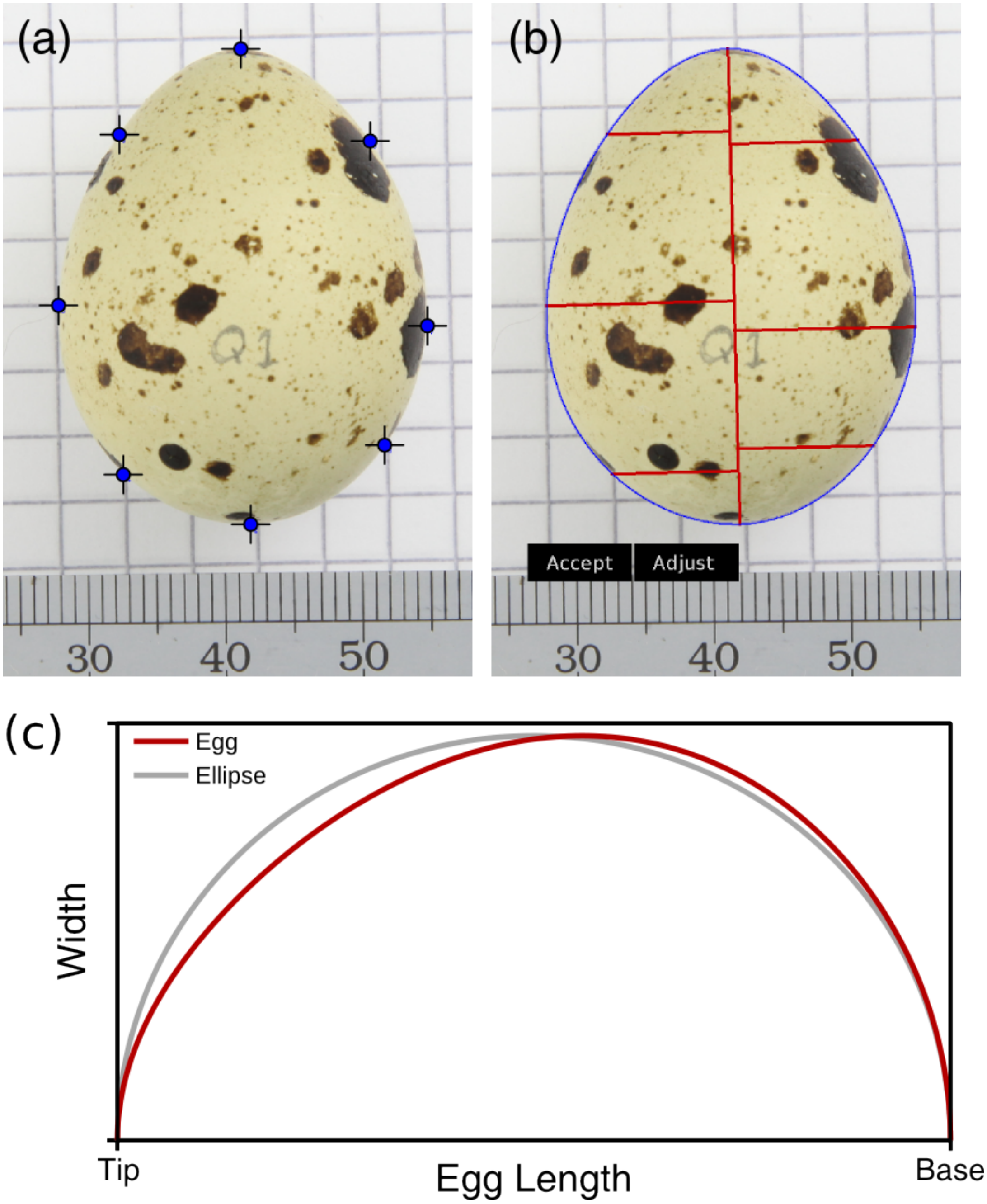

Figure 2. Example of egg shape fitting: Photograph of a quail egg (A) with eight points selected around it's periphery using ImageJ's multipoint selection tool. The tip and base of the egg must be selected, but the precise locations of the remaining points is highly flexible. The order in which points are selected does not matter. Panel (B) shows the egg shape model fit for user scrutiny. If the quality of the fit is suitable the user can click on the 'accept' button, or click 'adjust' to fine tune the anchor points. The code also presents a graph (C) of the egg's deviation from an ellipse (the basis of the 'pointedness' value – see methods and equation 2).

RESULTS

The egg shape model described here was able to fit a wide range of eggs with a mean R^2 value (±1 standard deviation) >0.999 (± 4.3×10^{-4}) the lowest R^2 value was 0.997, and the use of 48 anchor points resulted in volume estimates that had an average absolute difference of 0.20% (± 0.16) compared to estimates that used just eight anchor points (n = 103 eggs, 8 species). Increasing the number of anchor points did not significantly affect volume estimates (paired t-test: t = -0.699, P = 0.486). Photograph-based volume estimates using eight anchor points did not differ significantly from real egg volumes (t = *-0.862*, P = 0.392, mean absolute error = 0.73% (± 0.63), n = 72 eggs, 3 species), nor did Hoyt-based volume estimates (using shape coefficients that best-fit these data, see methods) (t = 0.283, P = 0.778, mean absolute error = 0.96% (± 0.74)). Egg shape coefficients for each species were calculated as: quail: 0.508; duck: 0.518; and chicken: 0.518. Pointedness was a significant predictor of error in Hoyt-based estimates (linear model $F_{3,68}$ = 7.32, P < 0.001), but not for photograph-based estimates ($F_{3,68}$ = 0.69, P = 0.441), see figure 3.

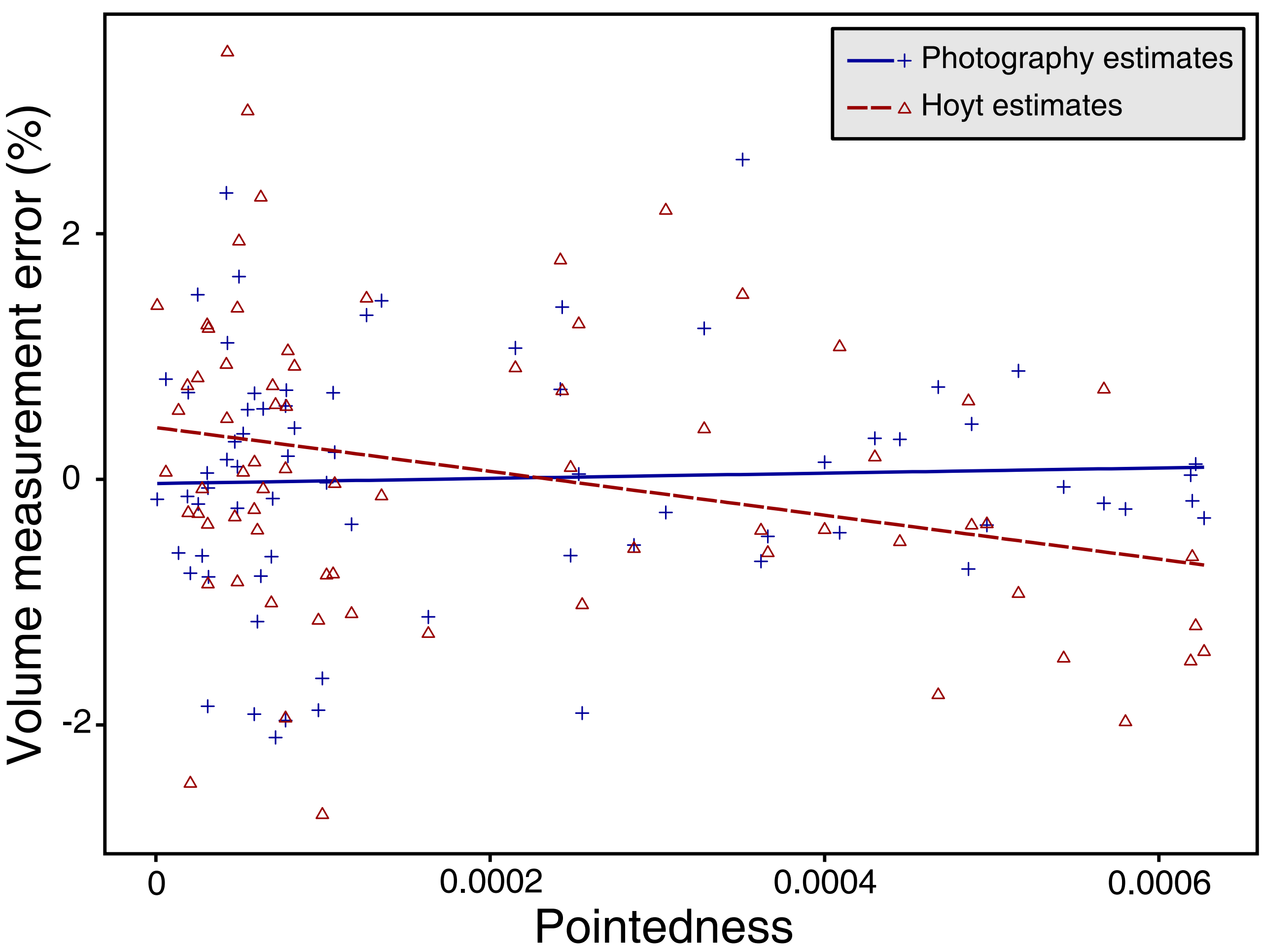

Figure 3. The effect of egg pointedness on measurement error: Photograph-based egg volume errors are not affected by egg pointedness, meaning intraspecific volume estimates can be relied upon irrespective of egg shape. In contrast, Hoyt-based (length and width) volume errors are significantly affected by pointedness, overestimating the volumes of elliptical eggs and underestimating the volumes of pointed eggs.

DISCUSSION

The egg shape model and fitting tools I present here provide a simple, user-friendly means for calculating various egg metrics, including volume, surface area and shape. Previous studies that have presented techniques for modelling egg shape from photographs have failed to test their models against real egg volumes in order to establish accuracy, instead pitting them against length and breadth-based models (e.g. Bridge *et al.*, 2007, Mónus & Barta, 2005). The technique presented here generated volume estimates that were not statistically different from real egg volumes, and were on average within 0.73% of real egg volumes; indeed, a portion of this variance must represent weight measurement error. Moreover, there was no evidence that egg shape affected the ability of this photograph-based technique to measure volume, implying it can be used to model inter and intraspecific egg shape variation. Hoyt-based volume estimates were not as accurate in predicting egg volume, and were dependent on the pointedness of the eggs and hence the original test population used to generate the species-specific variables.

The precision of this egg modelling technique was assessed by comparing model fits based on a large number of anchor points to models using the minimum of eight anchors per egg. There were negligible differences in estimated egg volumes when the minimum number of anchor points were used, and the mean R^2 value was greater than 0.999, implying the fit was near-perfect. There are a number of potential sources of error to consider when using this technique, the camera must be viewing the egg at right angles to it's long axis, failure to do so would result in shorter than expected lengths. A 100mm equivalent lens would be recommended to allow the camera to be positioned further away than a wide-angle lens, reducing perspective-based error. Lens distortion (e.g. barrel distortion) could adversely affect measurements. Lens distortion can be tested for by photographing a grid (e.g. graph paper) and ensuring the lines in the photograph are reproduced as straight lines. Lens distortion can be corrected for in a number of photo editing tools prior to egg measurements if deemed necessary. The scale bar should be placed level with the centre of the egg so that it is in the same focal plane as the centre of the egg and is subject to the same perspective, alternatively, the length of the egg (if known) can be used in lieu of a scale bar. Small apertures are recommended (e.g. f/8 or smaller) to maximise focal depth. Once anchor points have been selected around the edge of the egg the user is given an opportunity to visually inspect the quality of the fit and accept it or go back to adjust the anchor points, minimising the likelihood of poorly placed anchor points having a detrimental effect (see figure 2b). Unlike alternative methodologies (e.g. Bridge *et al.*, 2007), the eggs' patterns, the background used to photograph the eggs, and lighting conditions will not detriment or preclude analysis. Additionally, the model can be used to cut out eggs from their backgrounds or easily select an egg-shaped area, a feature useful for the growing number of studies using visual modelling of egg properties (e.g. Spottiswoode & Stevens, 2010, Stoddard & Stevens, 2010, Lovell *et al.*, 2013). The flexibility of the shape model could also be suitable for measuring the volumes of other elliptical objects, such as insect eggs that are difficult to measure by weight, but can be photographed easily using microscopy. Finally, the code for implementing this egg modelling technique is written as a plugin for ImageJ (Schneider *et al.*, 2012), a free and open source scientific image manipulation package. The script can be downloaded freely from http://www.jolyon.co.uk/research/eggs/.

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